# a method for transmission usaged cost in combined pool \& bilateral electricity markets 


#### Abstract

This article proposed new method for transmission cost Allocation in mixes pool and Bilateral Markets. In this method that is based on power flow equations and the real amount of usage from each of transmission line contributors and it can become application method in modern power markets. This method can be used in demand response in smart grids.At first, we compare, for only pool markets, this method for test and with results that got in one 4 bus network before. Then, in going on, Equations, he'd been written for market with bilateral contract, too at last, we will take one mixed market and will test equation for mixed market state and in a standard 24 bus network. IEEE-24bus RTS and we will analysis results.


Key words: power market- transmission cost- pool based markets- Bilateral markets - cost allocation.

## Introduction:

Main course of power industry in countries is recurrence of electrical energy demand for customers. From when that power networks spread for transference of electric energy that change to modern collective networks, gradually, demand to recurrence of power with higher quality and fewer expenditure of primary needs of customers. So, this order electric energy changes as a basic object. With naming of object to this energy, competition for purchase and sale of that increased and the basis of emergence of monopoly and create of power market for countries be prepared. Results outcome of structure renewal or deleting of provisions in different counties was different. But in a general view of create power markets, evaluate positive. Deleting of provisions in power industry necessitate that production parts, transference and distribution, preside independent of each other but, finally in all of network by one independent part like as independent beneficiary of net [ISO] operate. So expenditures of every part are independent from to another part, too. Spatiality of expenditure in transference part is a chief part of these expenditures. Specialty of this expenditure and it's justly receipt from net users in the all of countries is basic anxieties of this part in power markets.

The first step in calculation base of transference expenditure is that for transference line users are producers and consumer's expenditure as amount of every one of line uses had been divided. Different methods for example, megawatt mailer, contract line, post stamp and etcetera had been presented. That every method has defects and advantage. In this article, present new method that is base of bar diffusion equations that stock of every bus in power system determines potency for transference in every line, then divide this share between transference net users (generators and bars) fairly. For this aimed, proposed method at first on one market only with pool utilization and 4 bus, had been lest and compared these results with previous results in this system, then for one system with bilateral changes, had been written this equations and the slare of this usage from this market for buses deter mine and finally we, analysis production equations on one standard 24 bus system of power IEEE24bus RTS that include, pool market and presumptive bilateral contracts. Had been seen, we test them and analysis results.

## (I) load flow:

For every too market model, we need to $\pi$ model, them, it obligatory that apparent power of line have to determine , for, this aimed, we use from alloy diffusion equations for one transference line with $\pi$ model.

Apparent power flow for connection $L$ line between to bus i and j is beneath state:
$S_{i j}=\mathrm{V}_{i} I_{i j}^{*}$
A cording to fig 1 if vi and j respectively was criterion of line L and yio was sustenance of super line, we can write flow connection in beneath shape:

$$
\begin{equation*}
\mathrm{I}_{i j}=y_{i j}\left(\mathrm{~V}_{i}-\mathrm{V}_{j}\right)+y_{i o} \mathrm{~V} \tag{2}
\end{equation*}
$$

With subrogating connection 2 in connection 1 we have:

$$
\begin{align*}
S_{i j} & =V_{i}\left[y_{i j}^{*}\left(V_{i}^{*}-V_{j}^{*}\right)+y_{i o}^{*} V_{i} V_{i}^{*}\right]  \tag{3}\\
S_{i j} & =V_{i} y_{i j}^{*}\left(V_{i}^{*}-V_{j}^{*}\right)+y_{i o}^{*} V_{i} V_{i}^{*} \tag{4}
\end{align*}
$$

If we suppose $K$ in bus number, with writing voltage equation in bus (i), we have:
$\mathrm{V}_{i}=\sum_{k=1}^{n} Z_{i, k} I_{k}$
We can write injection flow to bus in beneath state with notice to passer potency of line.
$\mathrm{I}_{k}=\left(\frac{S_{k}}{V_{k}}\right)^{*}$
With determining bus flow in connection 6 and superpose that in connection b , voltage equation in group I had been rewritten as equation 7 and 8 .

$$
\begin{align*}
& \mathrm{V}_{i}=\sum_{k=1}^{n} \mathrm{Z}_{i k}\left(\frac{S_{k}}{V_{k}}\right)^{*}  \tag{7}\\
& \mathrm{~V}_{i}=\sum_{k=1}^{n}\left(\frac{\mathrm{Z}_{i k}}{V_{k}^{*}}\right) S_{k}^{*}=\sum_{k=1}^{n} x_{k} S_{k}^{*} \tag{8}
\end{align*}
$$

And in resembling state for bus j we have:
$\mathrm{V}_{j}=\sum_{k=1}^{n}\left(\frac{Z_{j k}}{V_{k}^{*}}\right) S_{k}^{*}=\sum_{k=1}^{n} w_{k} S_{k}^{*}$
With superposing connection 8 and 9 in power connection (equation 4), we rewrite pretence potency in equation in line.

$$
\begin{equation*}
S_{i j}=\left[y_{i j}^{*} \sum_{k=1}^{n} x_{k} S_{k}^{*}\right]\left[\sum_{k=1}^{n} x_{k}^{*} S_{k}-\sum_{k=1}^{n} w_{k}^{*} S_{k}\right]+y_{i o}^{*}\left[\sum_{k=1}^{n} x_{k} S_{k}^{*}\right]\left[\sum_{k=1}^{n} x_{k}^{*} S_{k}\right] \tag{10}
\end{equation*}
$$

For easiness in calculations. We write connection 80 as from of mat rise.
$S_{\mathrm{L}}=\left[\left(\mathrm{S}_{\text {bus }}^{*}\right)^{\mathrm{T}} D_{\mathrm{L}} \mathrm{C}_{\mathrm{L}}^{\mathrm{T}} \mathrm{S}_{\text {bus }}\right]\left[\left(\mathrm{S}_{\text {bus }}^{*}\right)^{\mathrm{T}} E_{\mathrm{L}} \mathrm{A}_{\mathrm{L}}^{\mathrm{T}} \mathrm{S}_{\text {bus }}\right]$
In connection 11, we have beneath supposition $\mathrm{S}_{\mathrm{L}}$ : apparent power in line L
$S_{\text {bus: }}$ : (difference of production power in bus and consumption power in same bus) injection power.
$D_{\mathrm{L}}: y_{i j}^{*} x_{k}$
$C_{\mathrm{L}}: x_{k}^{*}-w_{k}^{*}$
$E_{\mathrm{L}}: y_{i o}^{*} x_{k}$
$A_{\mathrm{L}}: x_{k}$
$F_{L}: D_{\mathrm{L}} \mathrm{C}_{\mathrm{L}}^{\mathrm{T}}+E_{\mathrm{L}} \mathrm{A}_{\mathrm{L}}^{\mathrm{T}}$
Injection power $S_{\text {bus }}$ organized from two veritable and fictitious powers. We can write that on basis of veritable power of bus.
$\mathrm{S}_{\text {bus }}=\mathrm{P}_{\text {bus }}+j Q_{\text {bus }}=\left(1+\mathrm{j} \frac{\mathrm{Q}_{\text {bus }}}{\mathrm{P}_{\text {bus }}}\right) \mathrm{P}_{\text {bus }}$
$S_{\mathrm{bus}}=\sigma . \mathrm{P}_{\mathrm{bus}}$

With substitution 6 and $\mathrm{F}_{\mathrm{L}}$ in connection 11, we have:
$S_{\mathrm{L}}=P_{b u s}^{T} \sigma^{*} F_{L} \sigma \mathrm{P}_{\text {bus }}$
$S_{\mathrm{L}}=P_{\text {bus }}^{T}\left(\sigma^{*} F_{L} \sigma\right) \mathrm{P}_{\text {bus }}=P_{\text {bus }}^{T} G_{L} \mathrm{P}_{\text {bus }}$
In equation 15 , pretence potency in transmission line had been get.
(II): calculation of expenditure of using of every bus from Trans mission line:

In this part, we have got the usage portion of every bus from (transmission line because buses include, market part ici pant are alloys and generator now, for this object, and we can spread connection 15 as a beneath shape:

$$
\begin{equation*}
P_{b u s}^{T} G_{L} P_{\text {bus }}=\sum_{i=1}^{n} \sum_{j=1}^{n} G_{L, i j} P_{i} P_{j}=\sum_{i=1}^{n} G_{L, i i} P_{i}^{2}+\sum_{i=1}^{n} \sum_{j=1}^{n} P_{i} P_{j} \tag{16}
\end{equation*}
$$

With spreading connection 16 we can get portion of every bus $(\mathrm{K})$ in transmission of potency from line L:
$S_{L, k}=G_{L, k k} P_{k}^{2}+\sum_{i=1}^{n}\left(G_{L, i k}+G_{L, k i}\right) P_{i} P_{k} \times \frac{\left|P_{k}\right|}{\left|P_{k}\right|+\left|P_{i}\right|}$

After calculation of appropriate portion amount to every bus, we have to calculate its expense:
$C_{L, k}=x_{L} \times S_{L, k} \times 1000$
$\mathrm{G}_{\mathrm{L}, \mathrm{K}}$ : $\mathrm{K} \quad$ the outlay usage of line L , appropriated to bus
$\mathrm{X}_{\mathrm{L}}: \quad$ Reactance of line L
H's normal that in calculation of expense for transmission line, we have to multiply the amount of bus in thousand Reactance of that line.
(III): specialty of transmission expense of one power 4 bus system on 4 pool market:

For correctness of mentioned connections from reference [1], we elect one power 4 bus system, that get in above reference replies we compare them.


Figure number 2: power 4 bus system

Proposed method on this system, we use fig number 2 and finally, for every 5 line , we get portion of buses and compare as table number 1 that get for this 4 bus power poll system from before methods.

Foe example, we bring line 2 in table number1:

| Bus | Expense (Dollar on hour) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | Proposed <br> method | $Z_{\text {bus }}$ | $Z_{\text {bus }}^{\text {avg }}$ | EBE | PS | PR |  |
| 1 | 40.77 | 35.9 | 26.92 | 28.3 | 39.4 | 24.8 |  |
| 2 | 15.16 | 21.4 | 11.74 | 20.2 | 9.1 | 23.8 |  |
| 3 | 29.35 | 31.8 | 46.6 | 36.4 | 32.8 | 24.2 |  |
| 4 | 11.72 | 7.9 | 11.47 | 12.1 | 15.7 | 24.2 |  |
| total | 97 | 97 | 97 | 97 | 97 | 97 |  |

Table number 1: specialty of transmission expense on every bus in line number 2.
After calculate the expense of transmission usage for every line, we get sum total buses expense from all lines in table number 2.

| Bus | Expense (Dollar on hour) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | Proposed <br> method | $Z_{\text {bus }}$ | $Z_{\text {bus }}^{\text {avg }}$ | EBE | PS | PR |  |
| 1 | 134.79 | 128.3 | 114.06 | 115.6 | 94.9 | 123.9 |  |
| 2 | 163.23 | 183.4 | 129.09 | 126.9 | 147.6 | 118.5 |  |
| 3 | 87.47 | 78.3 | 131.44 | 126.9 | 105.9 | 121.3 |  |
| 4 | 99.51 | 95 | 110.41 | 115.6 | 136.6 | 121.3 |  |
| total | 485 | 485 | 485 | 485 | 485 | 485 |  |

Table number 2: speciality of expense of sum total lines for every bus.

We saw in table number 1 that line number 2 in method PR every 4 bus have. Same payment value that not fair, in method PS and EBE these results are more fair because buses I and 3 have direct role, and have major portion of payment . In methods $Z_{\text {bus }}$ and $Z^{\text {avg }}{ }_{\text {bus, }}$ difference is that in method $Z_{\text {bus }}$ direction of price direction to producers but in $\mathrm{Z}^{\text {avg }}{ }_{\text {bus }}$ increase of price direction to consumers in proposed method according to table, results became more optimized and with numerical comparing we can result correctness of this proposed method. And numerical different between buses 1 and 3 designation of expense on every bus had been calculated that the tot payable of buses in proposed method with another methods are identical and equal with 485 dollar on hour and from numerical viewpoint its very near with final method of $\mathrm{Z}_{\mathrm{bus}}$. And results head had been getting with certifiable exactitude and integrity.

## Calculate of transmission expense in market with bilateral contracts

At first we have to get power due to bilateral contracts in buses on one power matrix $P_{5}$ that include information of buses and quantity of contracts.

If power system includes $n_{\text {bus }}$ and $m$ bilateral contract, $\mathrm{P}_{5}$ have to include 2 matrixes that one of them include connection between system buses in bilateral contract. Then one matrix T with dimensions mend MX1, we define that its elements are bilateral contracts power amount. We give name next matrix $M$ that its demesnes have to mxn. That condition prepare to matrices beat. Then:
$\mathrm{P}_{\mathrm{B}}=\mathrm{M}^{\mathrm{T}} . T$
Matrix M transmits information of buses with bilateral contract in system to $\mathrm{P}_{5}$.
For elements of matrix M , if exchange is from bus j to, bus I . Mij=1 and if it is converse $\mathrm{mij}=-1$ and if exchange in that bus or in another buses, we superpose $\mathrm{mij}=0$. With have connection 19 and superpose it in connection 15, we have.
$S_{\mathrm{L}}=P_{\text {bus }}^{T} G_{L} \mathrm{P}_{\text {bus }}=\left(\mathrm{M}^{\mathrm{T}} \cdot T\right)^{\mathrm{T}} G_{L} \mathrm{M}^{\mathrm{T}} . T=T^{T}\left(M . \mathrm{M}^{\mathrm{T}} G_{L}\right) T$
$S_{\mathrm{L}}=T^{T} \eta T$
With spread connection 21 , we can write connection 22,
$S_{L}=\sum_{i=1}^{m} \sum_{j=1}^{m} \eta_{L, i j} T_{i} T_{j}=\sum_{i=1}^{m} \eta_{L, i i} T_{i}^{2}+\sum_{i=1}^{m} \sum_{j=1}^{m} \eta_{L, i j} T_{i} T_{j}$
$S_{L, k}=\eta_{L, k k} T_{k}^{2}+\sum_{i=1}^{m}\left(\eta_{L, i k}+\eta_{L, k i}\right) T_{i} T_{k} \times \frac{\left|T_{k}\right|}{\left|T_{k}\right|+\left|T_{i}\right|}$
In connection 23 , specialty of portion to every bus ( $k$ ) in transmission line $L$ had been get, now for expense like as pool markets, we beat expense in 1000 retunes to getting transmission expense in this kind of market.

## Calculate of expense specialty for mixed markets (pool and bilateral)

For this state, at first, we have to change, buses potency, it meant that, buses power include two power, bilateral and pool can tract, then:
$\mathrm{P}_{\text {bus }}=P_{P}+P_{B}$
With superpose amount $\mathrm{P}_{\text {bus }}$ connection 24 in equation 15, we yield equation 25, 26.

$$
\begin{align*}
& S_{\mathrm{L}}=\left(P_{P}+P_{B}\right)^{T} G_{L}\left(P_{P}+P_{B}\right)  \tag{25}\\
& S_{\mathrm{L}}=P_{P}^{T} G_{L} P_{P}+P_{P}^{T}\left(G_{L}+G_{L}^{T}\right) P_{B}+P_{B}^{T} G_{L} P_{B}
\end{align*}
$$

And with superpose equivalent amount $P_{5}$ from equation 19 we can write equation 27, 28.

$$
\begin{align*}
& S_{\mathrm{L}}=P_{P}^{T} G_{L} P_{P}+P_{P}^{T}\left(\left(G_{L}+G_{L}^{T}\right) M^{T}\right) T+T^{T}\left(M G_{L} M^{T}\right) \cdot T  \tag{27}\\
& S_{\mathrm{L}}=P_{P}^{T} G_{L} P^{P}+P_{P}^{T} \lambda_{L} T+T^{T} \eta_{L} T \tag{28}
\end{align*}
$$

Phrase 28 divided 3 parts that first phrase is pretence power of pool market and second phrase include pool market and bilateral contract and third phrase include pretence power of bilateral contract market first and third terms of 28 equation phrase spread before $P^{T}{ }_{p} \lambda$ LT, we will spread that.
$\mathrm{P}_{\mathrm{p}}^{\mathrm{T}} \lambda_{\mathrm{L}} \mathrm{T}=\sum_{i=1}^{n} \sum_{j=1}^{m} \lambda_{L, i j} P_{i} T_{j}$
So, for one mixed market the portion of buses of pool and bilateral contract markets calculate in beneath shape:

$$
\begin{align*}
& \times \frac{\left|T_{k}\right|}{\left|P_{k}\right|+\left|T_{j}\right|} \\
& S_{B L, K}=\eta_{L, k k} T_{k}^{2}+\sum_{i=1}^{m}\left(\eta_{L, i k}+\eta_{L, k i}\right) T_{i} T_{k} \times \frac{\left|T_{k}\right|}{\left|T_{k}\right|+\left|T_{i}\right|}+\sum_{j=1}^{n} \lambda_{L, j k} T_{k} P_{j} \times \frac{\left|T_{k}\right|}{\left|P_{j}\right|+\left|T_{k}\right|} \tag{31}
\end{align*}
$$

| row | From bus no | to bus no | $\mathrm{P}(\mathrm{mw})$ |
| :---: | :---: | :---: | :---: |
| 1 | 7 | 10 | 80 |
| 2 | 1 | 3 | 100 |
| 3 | 21 | 20 | 80 |
| 4 | 2 | 5 | 100 |
| 5 | 13 | 13 | 50 |

Table3: Bilateral contract of 24 bus system IEEE

| Contract number | Expense (Dollar on hour) |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| Sum |  |

Table 4: the expense of usage portion from transmission line for bilateral line.


| Expense (Dollar on hour) |  |  |
| :---: | :---: | :---: |
| Bus number | Request | Producer |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |


| 7 |  |  |
| :---: | :--- | :--- |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 |  |  |
| 12 |  |  |
| 13 |  |  |
| 14 |  |  |
| 15 |  |  |
| 16 |  |  |
| 17 |  |  |
| 18 |  |  |
| 19 |  |  |
| 20 |  |  |
| 21 |  |  |
| 22 |  |  |
| 23 |  |  |
| 24 |  |  |
| total |  |  |

Table 5: the usage portion expense from participants in power pool market. that sulk define bus K portion in line L in pool market and in similar shape 85 LK , the bus potion K from line L in market with bilateral contract had been said. For calculation expenses of two these markets the get portion amount beat in reactance line, 1000 double.

## Case study:

For performance a mixed market that include many bilateral contract and pool market we select one standard test system and in this article had been selected power system , fig number 3(IEEE-24 bus RTS) that market in before supposition, had been utilize teal pool shape that generators and alloys have determined production and expenditure amount. Now we add 5 presumptive contracts to this market that we bring in table number 3 , theirs characteristic and contracts amount:
The amount of production increase and alloy due to this contract with regard to buses limitations, adding to system, now for 24 bus system, had been yield 2 tables 4 and 5, one table buses portion sue to power pool equation and another portion of every contact from transmission line in lateral changes.
In buses that is production and use coincides, production portion is production X bus expense and reagent portion of total expense is equal with unset production bus expense that is fairest payable portion that every actor in net pay expense in amount
of his use from net. Table 4 determines expense of every bilateral contract for transmission line and in altogether. Total expense connect to this market determine contrary to markets mixed. For contract number 5 that production and use are in one bus, we take expense equal with zero. In next table, number 5 proprietary expense of every of every bus in pool market for all line had been calculated and divided that for every bus, between generators and alloys and for inter mediate busses, that include buses $24,11,12, \ldots$ we didn't table take any expense that marker of correctness of connection.

## Deduction:

In this article, had been proposed new method in specialty of trans mission expense for mixed markets include power pool and bilateral contracts that at first compare for pure test pool market with before methods results that is very fairy and then had been written connection for market with bilateral contract and in last part for on mixed market in coincide shape, and then we saw produced connection on one standard system IEEE-24 bus RTS market in mixed shape and defined 5 presumptive. Contract, too. Then produced results were very rational and they use in application shape in power market with notice to correctness of connections and their precision.

